MŰEGYETEM 1782

**Budapest University of Technology and Economics**

Faculty of Architecture

Department of Mechanics, Materials and Structures

# Analysis and Design of Rocking Mechanisms

Thesis

**Tamás Ther**

Master of Science, Architectural Engineer (2010)

Supervisor:

László P. Kollár

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# Contents

# 1 Introduction

Historic masonry and stone buildings are vulnerable to earthquakes. Most of the churches built in Hungary in the XII-XIX$^{th}$ centuries contain stone or brick columns, walls and arches. Many of them were severely damaged by moderate ground motions. For example, in 1956 the vaults of a baroque church in Taksony was collapsed by the Dunaharaszti earthquake, M5,6 (Szeidovitz 1984). In the archive photos (Fig. 1) it is clearly visible, that the motion of the arches were so big that the vaults collapsed, while the arches themselves became seriously damaged but were not destroyed. This is the reason that in the investigation of stone or brick buildings both the stability of the structure and the motions during the excitation must be examined. It is also important to note that these structures were not designed for earthquakes, however today they must be investigated for the expected seismic event.

**Fig. 1** The ruined *Szent Anna* parish church after the earthquake in Dunaharaszti, 12$^{th}$ Jan. 1956 (Historia Domus 1956)

Static analysis of brick or stone structures are well explored and they are usually based on the thrust line analysis (see e.g. the fundamental paper of Heyman (1966)) with the aid of which a pushover analysis can also be performed. For earthquake design these methods are inapplicable, while these structures subjected to earthquakes show a clear size effect (the smaller the structure, the more vulnerable for earthquakes) which cannot be modelled with the static analysis (Housner 1963).

It is well known, that the classical analysis used for the design of regular buildings, such as the Response Modal Analysis (RMA) or even the time history analysis of elasto-plastic structures are not directly applicable for masonries, where the "rocking" of the blocks (opening and closing with impact) plays an important role in the nonlinear response of masonry structures (Makris and Konstantinidis 2003).

As a rule, we may say that there is no generally accepted method to analyze and design these kinds of structures.

In our thesis we make three important steps to reach a design methodology:

- modelling of single (rocking) blocks for earthquakes,
- modelling of columns consisting of rigid blocks, subjected to earthquakes,
- develop a design method to evaluate rocking structures.

## 2 Problem statement

*Modelling of single blocks*

Housner (1963) published his classical paper more than five decades ago, in which he presented a simple model for the rocking rigid block (Fig. 2). He determined the angular velocity after impact, $\omega_a$ (Fig. 2c) as a function of the geometry and the angular velocity before impact, $\omega_b$ (Fig. 2a):

$$\omega_a = \mu_{Hous}\omega_b, \qquad \mu_{Hous} = \frac{2h^2 - b^2}{2h^2 + 2b^2} \tag{1}$$

where $h$ and $b$ are the dimensions of the block (Fig. 2a), $\mu_{Hous}$ is the angular velocity ratio.

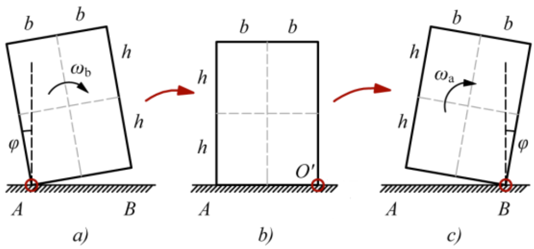


**Fig. 2** Housner's model for a rocking block

The rocking block was investigated experimentally by several researchers (Aslam et al. 1980; Lipscombe and Pellegrino 1993; Anooshehpoor and Brune 2002; Prieto-Castrillo 2007; Ma 2010). In almost every case, it was found that in the experiments the energy loss (and the decrease in angular velocity) is smaller than the one predicted by Housner's model (Fig. 3). The results are shown in Fig. 3.

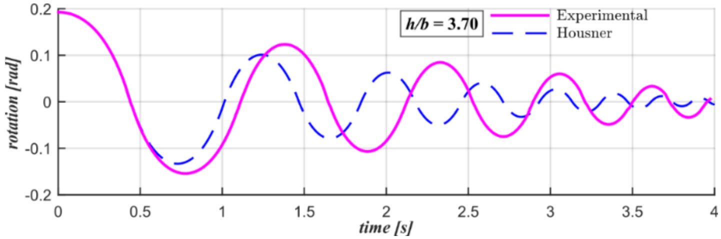


**Fig. 3** Typical time- displacement curve of a rocking block according to Housner's model (dashed line), and according to our experiment (solid line).

Note that in spite of the presented inaccuracies Housner's model is widely applied because of its simplicity and physical clarity. It can be a very important element of the analysis of structures subjected to earthquakes, where cracks may open and close during excitations. These are, for example: columns, walls and arches made of masonry, stone or unreinforced concrete blocks.

Researchers gave different explanations for the significant differences between the results of the experiments and the model, however no reasonable physical explanations were given.

Our aim is to give a physical explanation why Housner's model overpredicts the loss in energy, and to develop a physical model which agrees better with the experiments.

*Modelling of columns consisting of rigid blocks*

Masonry and stone columns are important structural elements. Their modelling must include the possible openings and closings of the cracks between the blocks, which require the use of an impact model.

For the impact of multi-block columns only a few mechanical models are available. Housner solved the single block, Psycharis (1990) presented a model for the two-block mechanism. As far as we know, no mechanical model of impact is available for columns with more than two blocks.

An alternative solution of the multi-block system is the discrete element method (DEM) (Winkler et al. 1995; Psycharis et al. 2000; Komodromos et al. 2008; DeJong 2009; Tóth et al. 2009; Dimitri et al. 2011; Lengyel and Bagi 2015). It can include the opening and closing of the interfaces and by setting certain parameters it seems a robust method for investigation multi-block columns. Using the discrete element method it was observed that monolithic blocks are more vulnerable to overturning than multi-block systems with the same overall dimensions (Psycharis et al. 2000; Dimitri et al. 2011).

Available FE codes (e.g. ANSYS, OpenSees, etc.) might be able to calculate the motion of blocks including the deformability of the elements and the geometrical nonlinearities. However, no proper “impact” and “opening” routines are available, hence these codes must be implemented.

We develop an own code with a low number of degrees of freedom, to obtain a robust, reliable tool to calculate the response of multi-block columns made of rigid blocks.

*Design methodology of rocking mechanism*

To evaluate the safety of the elements the overturning curve (OC) (acceleration as a function of duration) was introduced first by Housner (1963) for a half sine and a single rectangular pulse, then for other shapes by other researchers (Ishiyama 1982; Augusti and Sinopoli 1992; Anooshehpoor et al. 1999; Makris and Vassiliou 2012; Dimitrakopoulos and DeJong 2012; Voyagaki et al. 2013) and harmonic shaking by (Spanos and Koh 1985; Hogan 1992). Researchers investigated overturning also for earthquakes (Ishiyama 1982; Makris and Konstantinidis 2003; Peña et al. 2006; Peña et al. 2007; DeJong 2012; Makris and Vassiliou 2012; Voyagaki et al. 2013). Makris and Vassiliou (2012) showed that the effect of a near-fault, pulse-like earthquake can be replaced by a single rectangular pulse with properly chosen pulse duration. Ther and Kollár (2017a) have shown that fullness of the replacement pulse and the secondary pulse have a major effect on the OC.

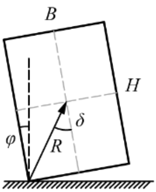


**Fig. 4** Geometry of the rigid block (the aspect ratio is: $H/B=\cot\delta$, moment of inertia about the corner point is $\Theta = \frac{4}{3}R^2 m$, where $m$ is the total mass)

For a given block and a given signal shape (e.g. a simple half sine) the OC can be defined as the curve which separates the safe and unsafe regions on the $a_p$, $t_p$ plane where $a_p$ is the maximum intensity of the main pulse lobe (acceleration) and $t_p$ is the duration of the pulse (Fig. 5a). If $a_p < a_{p,min}$ the block will not move at all, where (Fig. 4)

$$a_{p,\min} = g \tan \delta \tag{2}$$

and $g$ is the acceleration of gravity. Examples are shown for two and three consecutive half sines in Fig. 5b and c ($t_p$ is the duration of the half sine). All three figures show that for a given block both a shorter pulse with higher intensity and a longer pulse with lower intensity can cause the overturning of the block.

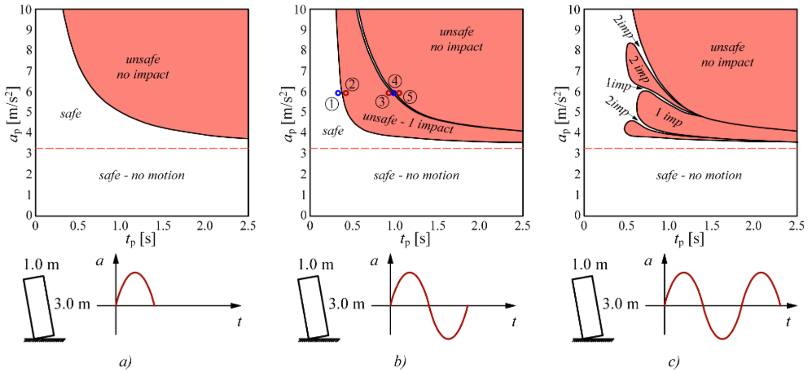


**Fig. 5** Overturning curve (OC) for a single block subjected to a half sine pulse (a) a full sine signal (b) and for a signal of three half sines (c)

We wish to develop a design methodology to determine whether the column is safe subjected to earthquake. We wish to determine the required design parameters (or curves) which can be applied for the checking of overturning of columns. We wish to give recommendations on how the earthquakes can be represented by a few parameters, in such a way that the responses of rigid columns calculated by time history analysis and by the developed procedures are close to each other, or at least the latter one can be used as a conservative approximation to predict overturning.

# 3 Method and modelling

## *Refinement of Housner's rocking model*

We apply a simple modification on Housner's classical model. It is assumed that the surface of the block (or the ground surface) is not perfectly smooth, but there is a small bump (or aggregate) in the middle (Fig. 6a). In this case the rocking occurs with two impacts. At impact the block rotates around point $C$. Following that a second impact occurs and the block rotates around corner $B$.

If the size of the bump (or aggregate) is small the time between the two impacts is also small, however, the final angular velocity is higher than in Housner's model.

If there are two bumps (Fig. 6b), rocking occurs with three impacts, and if there are $n$ bumps (which form a convex surface), rocking occurs in $n$+1 impacts (Fig. 6c). If the number of bumps goes to infinity, the block will "roll" and the energy loss is zero.

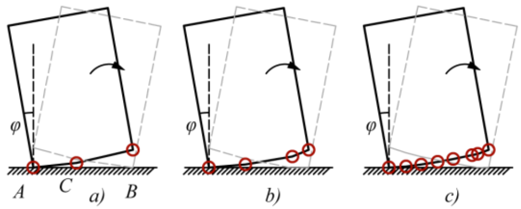


**Fig. 6** Rocking block. one bump in the middle (a), two bumps (b), several bumps (c)

It is assumed that the main reason that Housner's model overpredicts the loss in kinetic energy is that impact does not occur purely at the edges of the blocks (Fig. 7b), rather – in consecutive steps – at bumps and then at the edges (Fig. 7c).

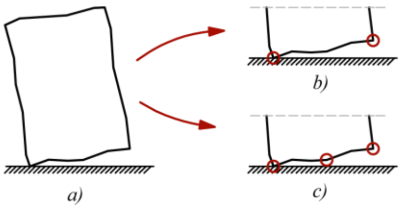


**Fig. 7** Comparison of Housner's model and the modification with an additional bump in the middle

To evaluate the above hypothesis experiments were carried out. Two granite blocks were manufactured with different aspect ratios, shown in Fig. 8. The rocking of each block was tested in 4 different configurations. Apart from the simple rocking (Fig. 9a), rocking on attached wires were investigated (Fig. 9b-d).

In case of configurations *b* and *d* two impacts occur during rocking; while in case of configuration *c* it was made sure that one impact occurs exactly at the chosen position defined by the wires close to the edges.

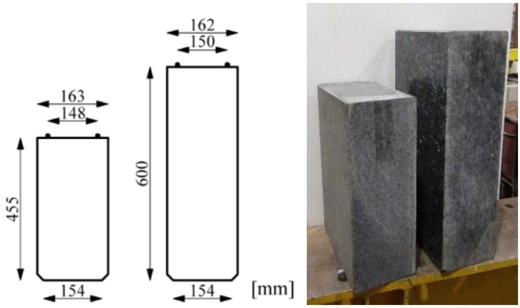


**Fig. 8** Picture and the sizes of granite blocks used in the experiments

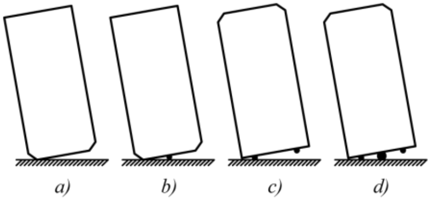


**Fig. 9** Configurations of a block applied in the tests

Typical example of the experiments is shown in Fig. 10. For configuration *a* the original Housner's model overpredicts the change in amplitude, and the modified underpredicts it (Fig. 10a).

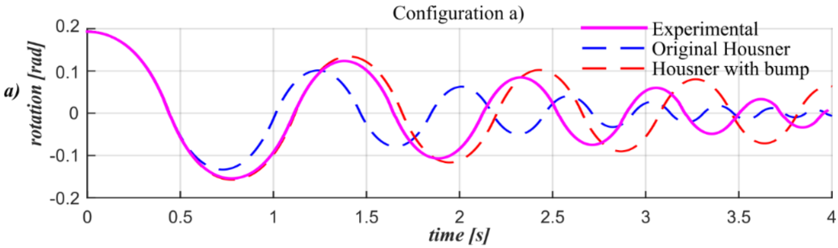


**Fig. 10** Example of the experimental results for configuration a) investigating the block with slenderness 3.7

We have also tested this hypothesis with the experiments published by Ogawa (1977); Aslam et al. (1980) and Prieto-Castrillo (2007). See Fig. 11, where the dashed line represents Housner's model with an extra bump in the middle.

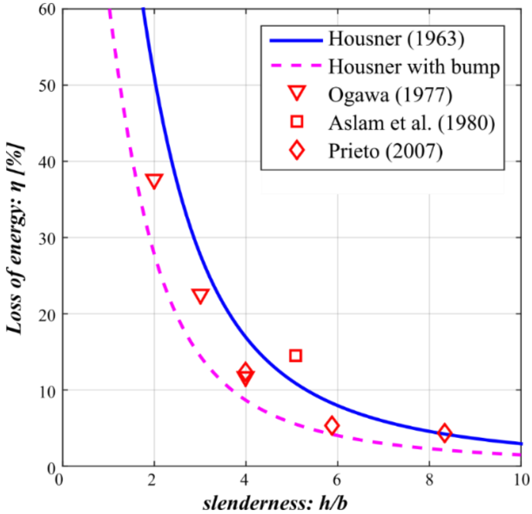


**Fig. 11** Experimental results compared with the classical Housner's model and with the refined model including a bump in the middle.

**Thesis 1.** A new physical explanation was given for the difference between the results of Housner's (1963) model and the experiments: Impact does not occur at the edges of the block, rather, due to the unevenness of the surface, with consecutive impacts **[1,2,3]**.

1.1 The above hypothesis was verified by experiments on granite blocks. When two steel wires were glued to the edges the experiments agreed well with Housner's prediction, while gave lower energy loss for three wires, or when no wires were introduced.

1.2 For numerical calculations it was suggested that impact is modelled by assuming two consecutive impacts, the first at the middle, and the second at the edge. This model gives similar results as most of the experiments reported in the literature (Ogawa 1977; Aslam et al. 1980; Prieto-Castrillo 2007).

## Model for multi-block columns

It is assumed that the column contains rigid elements and the motion occurs by the rotations at the cracked interfaces between two blocks. In the following, models are presented for the impact and opening of interfaces.

*The mechanical model*

We consider a multi-block cantilever structure where all the blocks are rigid. The adjacent blocks may move together (Fig. 12a) split open clockwise (Fig. 12b) or counterclockwise (Fig. 12c).

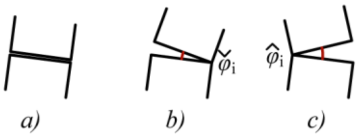


**Fig. 12** Opening possibilities of an interface (closed (a), open clockwise (b) and counterclockwise (c))

Since only one of the three cases may occur for every possible opening-configuration a new formulation is given. The key element of our model is that at the interfaces both clockwise and counterclockwise rotations are considered, where due to geometrical constraints

$$\check{\varphi}_i \leq 0, \quad \hat{\varphi}_i \geq 0. \tag{3}$$

For the formulation between impacts or openings only one of the three cases (Fig. 12) may occur in the same time and only one of each $\check{\varphi}_i - \hat{\varphi}_i$ pairs can be nonzero.

The equation of motion for the entire problem can be written as

$$\mathbf{M}_c\ddot{\boldsymbol{\varphi}} = \mathbf{m}, \tag{4}$$

where $\ddot{\boldsymbol{\varphi}}$ is second derivative of the rotation vector with respect to time, $\mathbf{m}$ is the load vector and $\mathbf{M}_c$ is the mass matrix.

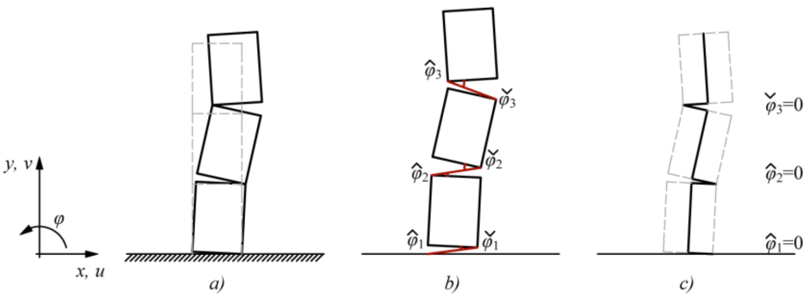


**Fig. 13** The degree of freedom of the model. One possible shape of a three-block system (a), the theoretically possible motions (b) and the choice of the zero and non-zero rotations (c)

In determining the response of the structure three tasks must be considered:

1) solving Eq.(4) for a given configuration,
2) solving for closing of one of the interfaces,
3) determining the opening of some of the interfaces.

*The impact model*

When an interface is closing impact occurs. An example is shown in Fig. 14, where impact occurs at the third interface; let us assume, that the first interface remains closed after impact, while the third and fourth one open up. The change in angular velocities is calculated as:

$$\begin{Bmatrix} \Delta\dot{\check{\varphi}}_2 \\ \dot{\check{\varphi}}_3 \\ \dot{\hat{\varphi}}_4 \\ \Delta\dot{\hat{\varphi}}_5 \end{Bmatrix} = \begin{bmatrix} m_{2,2} & m_{2,3} & m_{2,9} & m_{2,10} \\ m_{3,2} & m_{3,3} & m_{3,9} & m_{3,10} \\ m_{9,2} & m_{9,3} & m_{9,9} & m_{8,10} \\ m_{10,2} & m_{10,3} & m_{10,9} & m_{10,10} \end{bmatrix}^{-1} \begin{Bmatrix} m_{2,8} \\ m_{3,8} \\ m_{9,8} \\ m_{10,8} \end{Bmatrix} (-)\dot{\hat{\varphi}}_3. \qquad (5)$$

where $m_{ij}$ are the elements of the mass matrix $\mathbf{M_c}$.

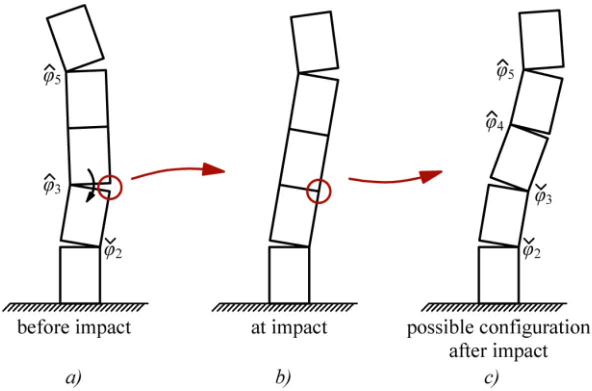


**Fig. 14** Configuration before (a), during (b) and after (c) the impact

Interfaces 3 and 4 open up, hence for these interfaces the changes in velocities are identical to the new velocities, while interfaces 2 and 5 are open before (and after) impact, for these interfaces Eq.(5) gives the change in speeds.

To choose the proper case is not an easy task, and it was found that in many cases the case, which seems trivial, is physically impossible. This is why we decided to investigate all the possible options.

Assume that there are $n^{\text{closed}}$ closed interfaces before impact. Each can be closed or open clockwise or counterclockwise after impact. This means that the total number of cases is

$$2 \times 3^{n_{\text{closed}}}. \tag{6}$$

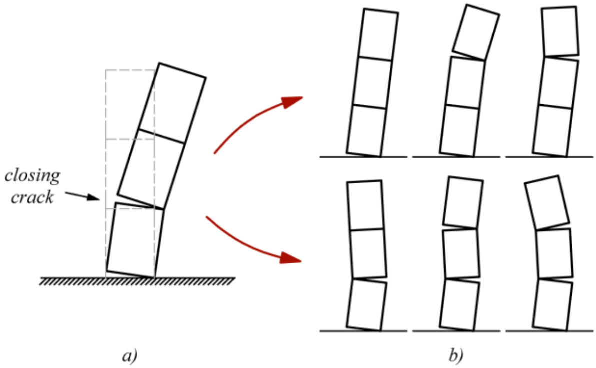


**Fig. 15** The possible rocking motions after impact ($n_{\text{closed}}$=1)

The following strategy is recommended:

- for each case determine the change in velocities during impact,
- throw out the impossible configurations (Eq. (3)),
- if there are more than one kinematically admissible configuration chose the one for which the kinetic energy ($E_{\text{kin}} = \frac{1}{2}\dot{\boldsymbol{\varphi}}^{\text{T}}\mathbf{M}_{\text{c}}\dot{\boldsymbol{\varphi}}$) is the highest, i.e. where the dissipated energy is the lowest.

The impact model was verified by the expressions presented by Psycharis (1990) for a two block mechanism.

*Model for opening*

If at one (or more) interfaces the eccentricity of the normal forces reaches the width of the column, one or more interfaces split open. Accordingly, the configuration changes and the equation of motion (Eq.(4)) must be solved with this new geometry.

It seems a good strategy to either open the interface where the eccentricity is the highest or to open all the interfaces where the eccentricities are outside the width of the blocks. We found, however, that this procedure can be numerically unstable. In theory there are 8 possible opening configurations for the problem presented in Fig. 16, however, the one shown in Fig. 16e will occur.

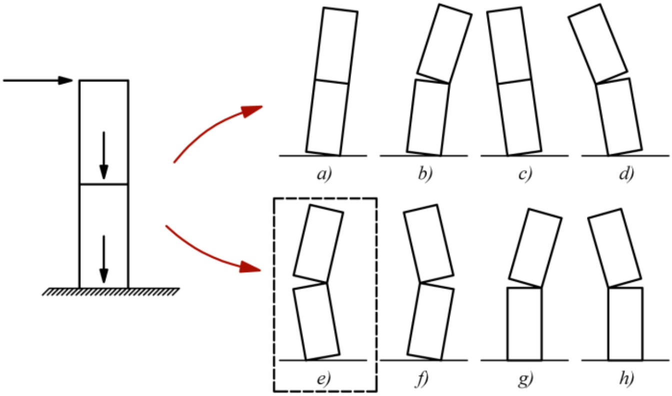


**Fig. 16** Possible openings of a two-block column

When the number of closed interfaces ($n_{\text{closed}}$) are not high, the recommended strategy is as follows:

- determine all the possible configurations ($3^{n_{\text{closed}}} - 1$),
- chose those where the displacements after the first time step are compatible, these are the kinematically admissible configurations,
- if there are more than one possible cases, chose the one where the kinetic energy is the highest.

When the number of the closed interfaces is high, the above procedure is very time consuming, and it is a better strategy to reduce the length of the time steps.

*Dynamical model for multi-block columns*

Several robust methods are available to solve Eq.(4), here Wilson's method (Chopra 1995) was applied. At every time step three conditions were investigated:

- at every closed interface the eccentricity of the normal force must be within the width of the elements,
- at every open interface the motions must satisfy Eq.(3),
- at every interface the normal force must be compression, and $\mu N \geq |V|$, where $\mu$ is the friction coefficient, $N$ is the normal force and $V$ is the shear force.

If either one of these is not satisfied the calculation is terminated. In the first case one (or more) closed interfaces must split open, in the second case impact occurs, while in the third case the column may disintegrate and the whole process is terminated.

*Experimental verification of the impact and opening model*

Two granite blocks were placed on top of each other (Fig. 17). The upper block was released from an inclined position and the rotation of the blocks was measured by x-IMU devices. The recorded and the simulated rotations of the blocks are presented in Fig. 18.

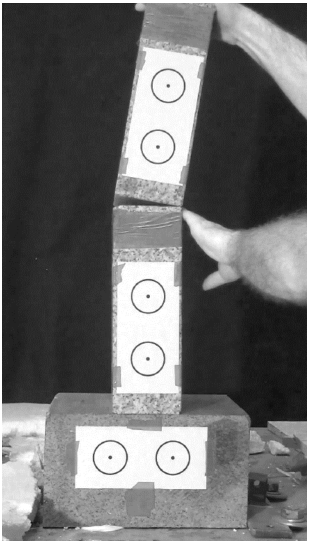

**Fig. 17** The initial inclination of the two-block system

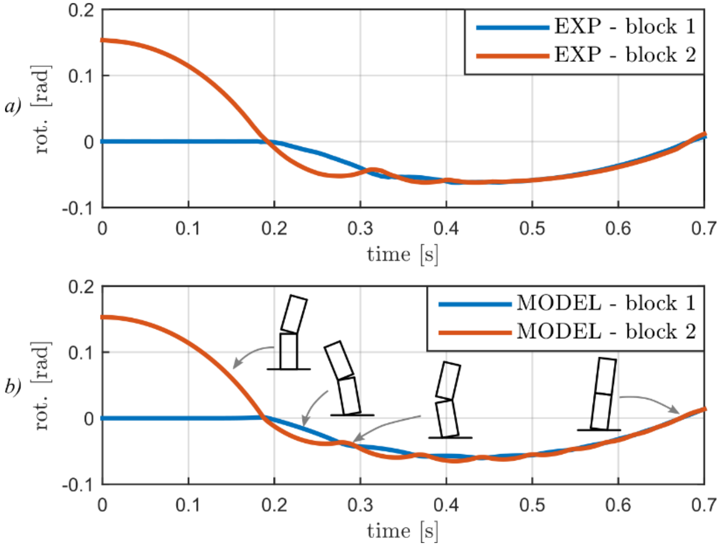


**Fig. 18** The recorded (a) and the simulated (b) rotations of the two-block system during the free-rocking experiments

The measured and the calculated rotations are close to each other. More importantly the observed and predicted opening-patterns are practically identical.

*Base excitation of multi-block columns*

A column made of 3 granite blocks was investigated (Fig. 19).

The system was placed on a shaking table and it was excited by a sine pulse with period 0.6 s, and amplitude 45 mm. The motions of the blocks and the shaking table have been recorded by a Full HD camcorder. The rotations have been identified by an image-procession algorithm, written by the authors. One of the experimental results is presented in Fig. 20a.

In Fig. 20b the calculated rotations of the three-block system is plotted. The same opening and closing schemes are clearly visible. The experimental and the numerical results show acceptable agreement.

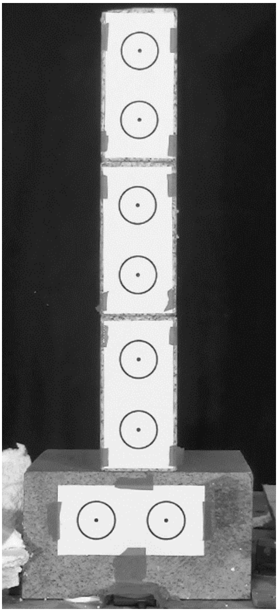

**Fig. 19** The three-block column made of granite blocks

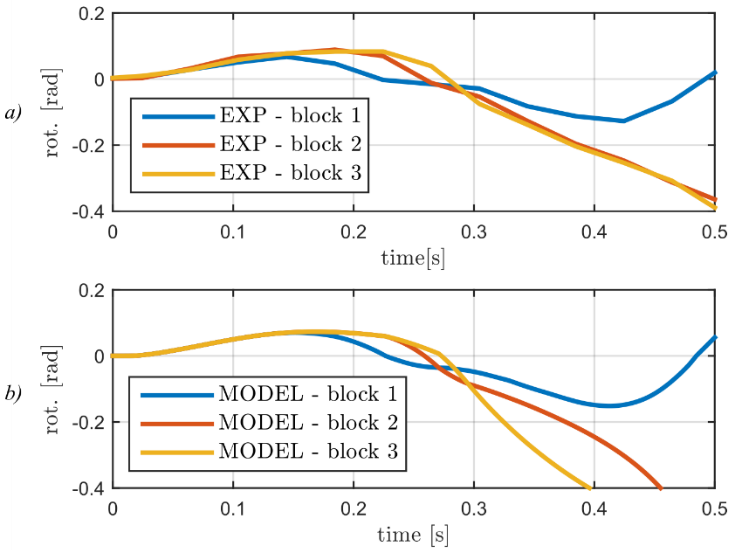


**Fig. 20** The experimental (a) and numerical (b) results of a base excitation test

**Thesis 2.** A new impact model is developed for 2D multi-block columns, which enable us to calculate the change of velocities during impacts. The key of the model that in formulating the problem at the closing interface both the clockwise and counter-clockwise rotations are taken into account, even though these motions exclude each other **[1,4]**. For one or two blocks this new model simplifies to those of Housner (1963) and Psycharis (1990).

**Thesis 3.** A model was developed to investigate the actual opening scheme of multi-block columns during impact. The kinematically admissible opening configurations are chosen by investigating the signs of post-impact velocities, and then the one is considered, where the kinetic energy is the highest (the energy loss is the lowest). The model was validated by experiments. **[4]**

**Thesis 4.** It was shown that simple opening of the interface where the thrust line is outside of the cross-section may be numerically unstable. A model was developed to investigate the actual opening configuration of multi-block columns. The kinematically admissible configurations are chosen by investigating the signs of displacements. The model was validated by experiments. **[4]**

**Thesis 5.** A new model is developed for rocking of multi-block columns, which contains an opening and an impact model, together with the improved Housner's model presented in Thesis 1.2. **[4]**

5.1 The model was verified by experiments.

# 4 Overturning of single blocks for base excitation

*Overturning Acceleration Spectra*

The *normalized overturning curve* (OC) (Housner 1963) is shown for a block with a given aspect ratio subjected to a single half sine pulse in Fig. 21a. Both axes are dimensionless, the vertical axis is normalized by $a_{p,min}$ (Eq.(2)), while the horizontal axis by the inverse of the "frequency parameter" ($p$), defined by Housner (1963):

$$p = \sqrt{\frac{mRg}{\Theta}} = \sqrt{\frac{g}{\alpha R}}, \quad \alpha = \frac{4}{3}, \tag{7}$$

where $\Theta$ is the mass moment of inertia about the corner point where the rotation occurs (Fig. 4), $m$ is the total mass, $R$ is the distance between the centre of mass and the corner point and $g$ is the acceleration of gravity.

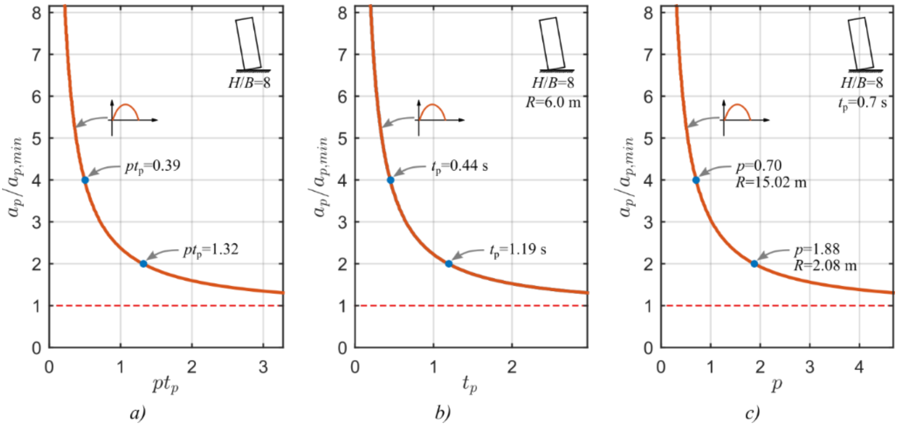


**Fig. 21** Normalized overturning curve (OC) for a single block subjected to a half sine pulse (a) (see Housner (1963)). The OC shows the effect of the pulse length (b) while the OAS shows the effect of the block size (c).

The plot in Fig. 21c is called *Overturning Acceleration Spectrum* (OAS), where the horizontal axis depends only on the block's size and not on the pulse duration.

*Overturning acceleration spectra of single blocks for earthquake excitation*

We define the *Overturning Acceleration Spectrum* (OAS) for an earthquake record as the curve which separates the safe and unsafe areas in the $a_p/a_{p,\min}$, $p$ coordinate system. An example is shown in Fig. 22a. The complexity of the earthquake record leads to several safe "bays" and "islands" within the unsafe region. The dimension of the horizontal axis is 1/sec.

We defined the *transformed OAS* in such a way that the horizontal coordinate of OAS (Fig. 23a) is multiplied by $a_p/a_{p,\min}$. The result for an earthquake record is shown in Fig. 23b, where

$$f = p\frac{a_p}{a_{p,\min}}. \tag{8}$$

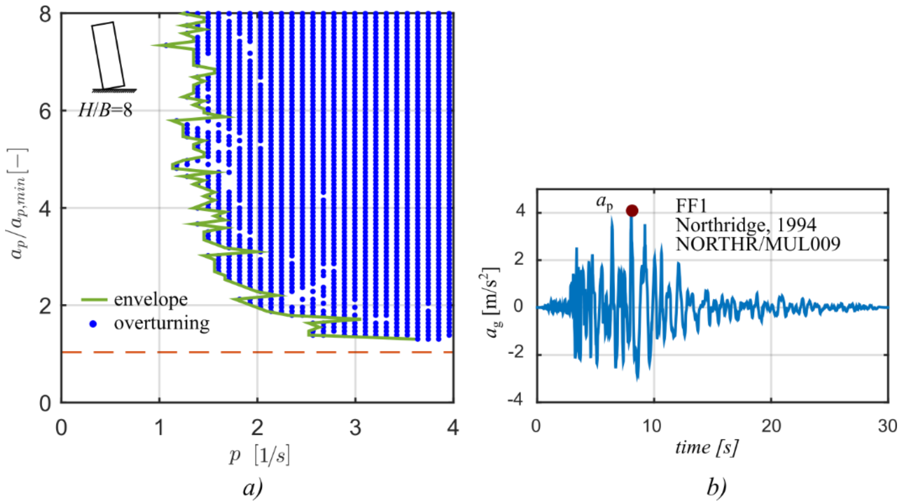


**Fig. 22** The *Overturning Acceleration Spectrum* (OAS) of a single block (dot represents overturning) (a) based on the Northridge-1994, NORTHR/MUL009 record (b)

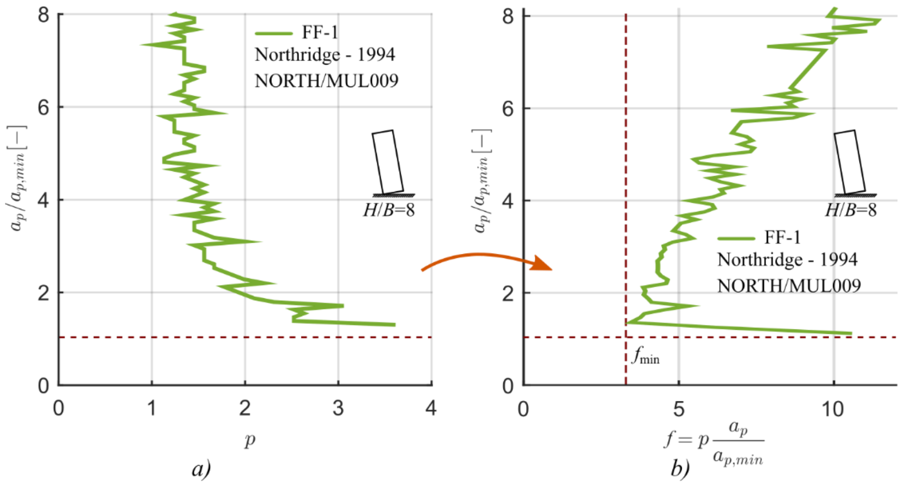


**Fig. 23** The OAS (a) and the transformed OAS (b) for an earthquake record (Northridge – 1994, North/MUL009 component)

# 5 OAS of multi-block columns

We investigate columns contain 2 or 3 blocks. The results are shown in Fig. 24 for $H$=12 m, $B$=1 m. In the plots the overturning of the structures are plotted on the $a_p$-$t_p$ plane. It can be seen that with reasonable accuracy single blocks are more vulnerable than multi-block columns.

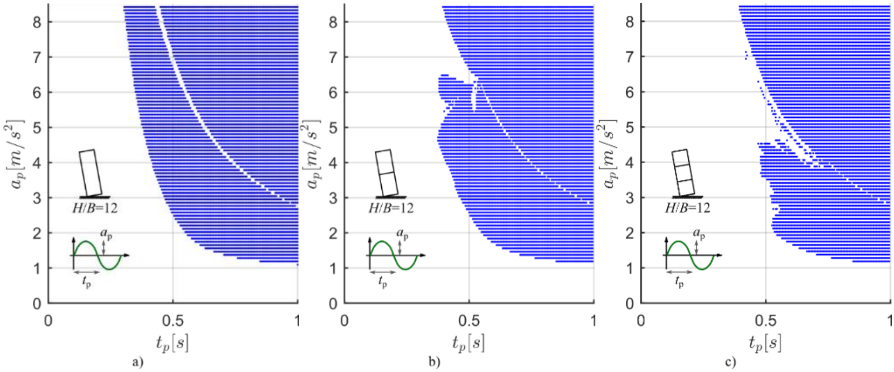


**Fig. 24** The overturning of columns (H=12 m, B=1 m) consisting of 1, 2 and 3 blocks subjected to full sine pulse. The dots represent overturning.

The effect of energy dissipation is shown in Fig. 25 for earthquake excitation. Three cases are presented:

1) 1 impact, which is identical to Housner's model (Fig. 7b),
2) 2 consecutive impacts, which is the recommended model, and which agrees well – for a single block – with the experiments (Fig. 7c),
3) 10 consecutive impacts, when there is practically no energy dissipation, the elements 'roll' on each other.

Note that the 1, 2 or 10 impacts are instantaneous, the total duration is zero.

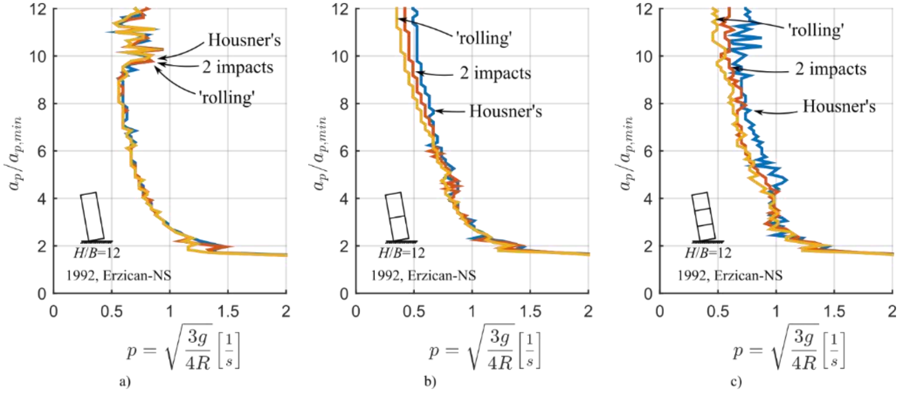


**Fig. 25** The OAS of columns consisting of 1, 2 and 3 blocks with different energy dissipations. The column is subjected to 1992, Erzican-NS earthquake record.

As a rule the lower the energy dissipation, the more vulnerable structures are to overturning.

We may observe that the more the number of blocks in a column the more important the effect of energy dissipation is. The explanation is that during rocking the energy dissipations of a stocky element is higher than that of a slender one (Housner 1963). It is also clear that the effect of energy dissipation is more important for earthquake records than for simple signals, the reason is that there are more impacts for a complex record than for a simple pulse.

**Thesis 5.**

5.2 Investigating columns for pulse-like signals and real earthquake records, it was found that monolithic blocks are more vulnerable for overturning than multi-block structures. **[4]**

5.3 In contradiction to monolithic blocks, it was found that for multi-block columns the dissipation of energy during impact plays an important role even for slender columns. **[4]**

# 6 Design method for rocking columns

We investigate the behavior of single rigid blocks, for it was shown, that single (monolithic) columns are more vulnerable, than multi-block columns.

The applicability of our approach was investigated numerically using time history analysis. In our research 56 near field (NF) and 44 far field (FF) records were considered, which are given in FEMA P695 (2009). We investigated 4 different aspect ratios ($H/B$=3, 5, 8 and 12), 80 different peak ground acceleration levels (from $a_{p,min}$ to 10 times $a_{p,min}$) and up to 120 different sizes (from $R_{max}$=1000 m down to $R_{min}$=0.1 m, always searching for the largest unsafe block size for a given peak ground acceleration).

*Characteristic Overturning Acceleration Spectra*

For a given location we can determine the OAS for several earthquake records (Fig. 26a) and then a statistically determined *characteristic OAS* for a given probability of exceedance can be defined (Fig. 26b). The determination of this *characteristic OAS* is not the subject of our research, we just give a theoretical curve in Fig. 26b. In the following calculation a reasonably high slenderness: $H/B$=12 will be used. Therefore, the resulting *characteristic OAS* can be used for a wide range of aspect ratios.

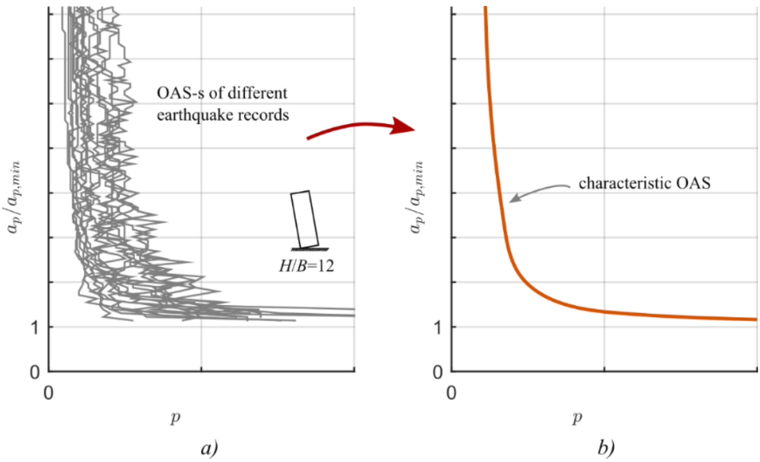


**Fig. 26** OAS-s at a given location (a) and the determined *characteristic OAS* (b)

*Simplified Overturning Acceleration Spectra*

Our aim is to have an OAS which can be represented by a few parameters. To reach this goal we consider the *transformed OAS* (Fig. 23b). For real earthquake records in most cases a vertical line at $f_{min}$ is considered to be a reasonable approximation, which is shown in Fig. 23b.

It is recommended that the transformed OAS is approximated by a horizontal and vertical line, by the *simplified transformed OAS* (Fig. 27). The horizontal location of the vertical line is given as a function of the normalized critical impulse:

$$f_{min} = \frac{i_{cr}}{t_I} = \frac{1}{t_I}\sqrt{\frac{2}{1+\cos\delta}} \approx \frac{1}{t_I}, \tag{9}$$

where $t_I$ is the *'replacement impulse duration'*. Its value must be determined numerically, using real earthquake records.

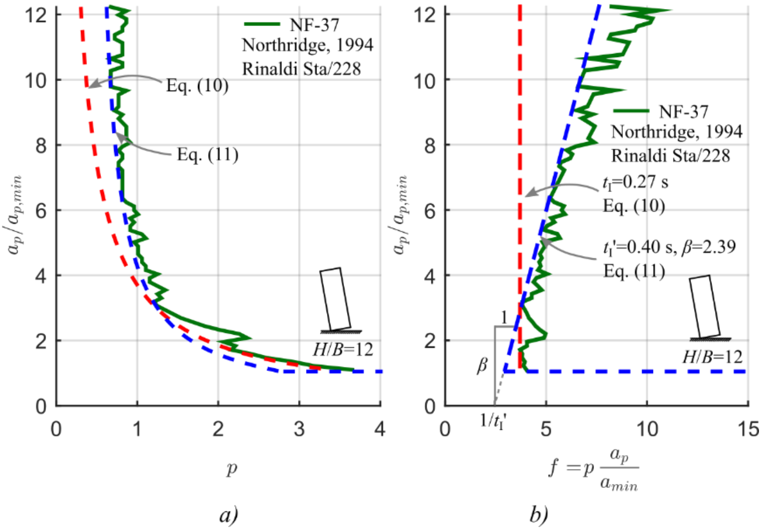


**Fig. 27** OAS (a) and transformed OAS (b) for the Northridge earthquake compared to simplified curves ($t_I$=0.27, $t_I'$ =0.40, $\beta$=2.39)

It is recommended to represent earthquakes by two parameters: $a_p$ and $t_I$. We may approximate the *transformed OAS* by an inclined line, which can be defined by two parameters: $t_I'$ and the slope of the inclined line, $\beta$.

On the basis of $t_I$ the *simplified OAS* can be calculated as (Eq. (8)):

$$\frac{a_p}{a_{p,\min}} = \max\left\{\frac{i_{cr}}{t_I}\frac{1}{p}, 1\right\} \approx \max\left\{\frac{1}{t_I}\frac{1}{p}, 1\right\}. \tag{10}$$

To achieve a better fit, we may approximate the *transformed OAS* by a vertical and an inclined line, which results in

$$\frac{a_p}{a_{p,min}} = \max\left\{\frac{1}{t_I}\frac{1}{p}; \frac{1}{t'_I}\frac{1}{p-1/\beta}; 1\right\}. \tag{11}$$

For the 100 investigated earthquakes we determined numerically the $t_I$, $t_I$' and $\beta$ values.

*The replacement impulse duration*

It is worthwhile to compare the calculated *replacement impulse durations* to the parameters of the main pulse lobes of real earthquake records. To reach this goal we made a very simple calculation shown in the followings.

Overturning may be caused by either a large acceleration ($a_{max}$) or by a large impulse ($I_{max}$), the corresponding pulses are shown by shaded areas in Fig. 28a. (They may coincide.) To capture both we defined a single pulse as a simple sine curve with $a_{max}$ and $I_{max}$ (Fig. 28b). The fullness of a sine curve is $F$=0.64, and its duration is:

$$t_p = \frac{I_{max}}{F a_{max}}. \tag{12}$$

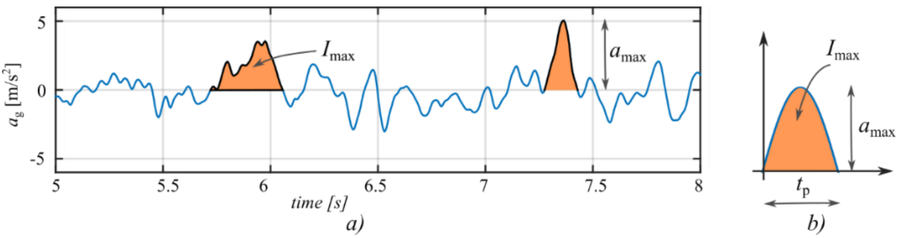


**Fig. 28** An acceleration record and the definition of $a_{max}$ and $I_{max}$,(a), and the replacement sine curve (b). The record is the 1979, Imperial Valley – Bonds Corner/140 (NF-3)

We investigated the correlation between $t_I$ and $t_p$, (Fig. 29), and found high correlation, higher for NF than for FF records. Interestingly, linear regression gives approximately $t_I \approx (0.8 \div 1)t_p$.

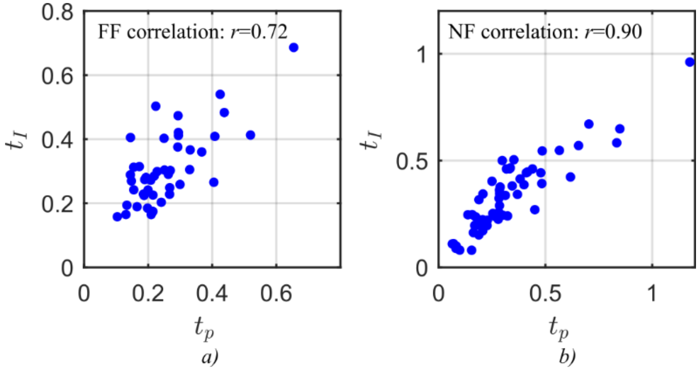


**Fig. 29** The corresponding $t_I$ and $t_P$ values for the FF (a) and the NF (b) records. The correlation coefficients are 0.72 and 0.90 for FF and NF, respectively

**Thesis 6.** The OC was generalized for earthquake records, and the *overturning acceleration spectrum* (OAS) was introduced. In principle a *characteristic OAS* can be determined by the statistical evaluation of time history analyses of rocking mechanisms for earthquake records at a given location. **[3,5]**

6.1 Based on the analyses of rigid blocks for 100 earthquake records, it was found that the OAS can be characterized well by one parameter: the "*replacement impulse duration*". It was also shown that the *replacement impulse durations* and the actual impulse durations of the main pulses of earthquake records are highly correlated. **[5]**

6.2 A simple design equation was recommended to determine the safety of structures subjected to earthquakes for overturning. **[5]**

# 7 Future works

As an extension of our column model, we are planning to develop a multi-block 2D arch model. In the literature, the masonry arch is investigated as a four-hinge mechanism for pulses and earthquake excitations (De Lorenzis 2007; DeJong et al. 2008; DeJong 2009). This system is a single degree of freedom structure.

Due to our preliminary calculations the locations and the number of open interfaces change during the motion of the structure which might influence the results. We plan to explore this question and to develop OAS-s for arches.

# 8 Main publications

*Publications on the subject of the theses*

[1] Ther T, Kollár LP. Response of Masonry Columns and Arches Subjected To Base Excitation. *In: Ansal A, editor. Second European Conference on Earthquake Engineering and Seismology, Istanbul: 2014.* DOI: 10.13140/2.1.3314.2086.

[2] Ther T, Kollár LP. Refinement of Housner's model on rocking blocks. *Bulletin of Earthquake Engineering 2017*; 15(5): 2305–2319. DOI: 10.1007/s10518-016-0048-8.

[3] Ther T, Kollár LP. Refinement of Housner's model and its application for the overturning acceleration spectra. *16th World Conference on Earthquake Engineering, Santiago, Chile: 2017.*

[4] Ther T, Kollár LP. Model for Multi-Block Columns Subjected to Base Excitation. *Earthquake Engineering & Structural Dynamics 2017.* DOI: 10.1002/eqe.2957 (accepted for publication)

[5] Ther T, Kollár LP. Overturning of rigid blocks for earthquake excitation. *Bulletin of Earthquake Engineering 2017.* (under review)

*Other publications*

[6] Ther T, Sajtos I, Armuth M, Strommer L. Ribbed vaults of the Nagyvázsony monastery church – Geometrical factor of safety highlights the secret. *Periodica Polytechnica Architecture 2010*; 41(1): 3. DOI: 10.3311/pp.ar.2010-1.01.

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